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Gravitational hedgehog, stringy hedgehog and stringy sphere

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Abstract: We investigate the solutions of Einstein equations such that a hedgehog solution is matched to different exterior or interior solutions via a spherical shell. In the case where both the exterior and the interior regions are hedgehog solutions or one of them is flat, the resulting spherical shell becomes a stringy shell. We also consider more general matchings and see that in this case the shell deviates from its stringy character.

Keywords: Spontaneous Symmetry Breaking, Classical Theories of Gravity.

Contents

1. Introduction

Topological defects may arise during phase transitions in the early Universe [1]. Depending on the type of the spontaneously broken symmetry, monopoles, strings, domain walls and textures may be produced [2]. Among these defects, cosmic strings are studied extensively. They have wider applications in galaxy formation and gravitational lensing theories which might have observational implications [2].

There are also some solutions which are constructed from ensembles of cosmic strings. The energy momentum tensor of an infinitely long straight cosmic string along z axis is given by

$$T_{zz} = -T_{00} = -\mu\,\delta(x)\delta(y)\,, \tag{1.1}$$

with other components vanishing [3]. The spacetime outside the straight cosmic string is flat but conical since the well known exterior metric of an infinitely long cosmic string contains an angular deficit. Namely in usual cylindrical coordinates:

$$ds^2 = -dt^2 + dr^2 + dz^2 + (1-4G\mu)^2 r^2 d\phi^2\,, \tag{1.2}$$

where the angular coordinate ϕ has the range $0 \leq \phi \leq 2\pi$ and μ is the mass per unit length of the string.

Since we will present a solution with an ensemble of cosmic strings, here we discuss some solutions of the Einstein equations giving rise to cosmic stringy structures. By cosmic stringy structures we mean a d dimensional hypersurface in 4 dimensional space-time ($d \leq 3$) such that on this hypersurface the principal stresses of the Energy-Momentum tensor satisfy

$$\sum_{i=1}^{d} p_i = -\rho\,, \tag{1.3}$$

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where ρ is the energy density and p_i are the principal stresses. The famous solution of Vilenkin [3] describing an infinitely long straight cosmic string is our first example of these structures where the string is taken as a δ-function source ($d = 1$). On the string condition (1.3) is satisfied with $p_z = -\rho$.

One example of the two dimensional stringy structures satisfying condition (1.3) is the stringy hollow cylinder solution where an infinitely long cylinder is constructed from straight strings [4]. In this solution, the interior region is flat and the exterior region is given by a locally flat cosmic string metric (1.2). This solution can be extended to the rotating hollow stringy cylinder case [5]. The solution we will present in section 3 is another example of cosmic stringy surfaces where strings lie on the surface of a sphere. Unlike the hollow cylinder case, either both or the interior or the exterior regions are not vacuum but contain radial strings.

The solutions with multiple parallel or nonparallel straight cosmic strings moving with different velocities are given in [6]. Thus using this solution, one can orient strings even non parallel. Using a different method, Arik et al. presented a solution [7] where a straight cosmic string decay into conical surfaces at its end points and ending up as a stringy sheet with radial strings. This solution can be extended to the other cases [8] including a string changing into a cylinder, a cylinder changing into a cylinder with different radius and a cylinder decaying into conical surfaces at its endpoints and ending up as a sheet with radial strings.

The Gott-Hischok-Linet interior solutions [9]–[11] are three dimensional examples of the cosmic stringy structures where a solid cylinder is filled with long string fibers with $p_z = -\rho$. In the first two solutions the energy density is constant in the cylinder whereas in the third one it is variable. Both solutions match to the string exterior smoothly. These are the first examples of stringy volumes.

One of the earlier solutions of the stringy volumes is given by Letelier [12]. The solution has a cloud of strings as a source lying between the Schwarzschild exterior and a fluid interior. The line element given in this solution also describes a global monopole (hedgehog) which we will review in the next section.

The coasting universe [13] is a cosmological solution where the universe is dominated by cosmic strings [14]. The equation of state of the fluid which fills up this universe is $p = -1/3\,\rho$. This can be thought to be made of randomly oriented straight strings.

Recently, another example of the stringy volumes and surfaces has been given in [15]. This is a solution with an interior with a string-like equation of state, a surface layer with again a string-like equation of state and a locally AdS exterior in toroidal coordinates.

Unlike straight strings, curved strings have nonzero gravitational potential [2]. Hence, they cannot be static. It is known that a loop of string may oscillate or, due to its large tension, either emits radiation and vanishes completely, or collapses and emits some portion of its energy as radiation and forms a black hole [16]. To have a loop stable one needs to support it again collapse. A circular string solution is given in [17] as an initial value problem. The solution we present in section 3 contains strings lying on a surface of a sphere. In our solution these strings are supported by radial strings.

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Actually, there could be more solutions that we are not aware of. However, the solutions we have mentioned reflect the richness of the situation of constructing more complex structures such as stringy surfaces or volumes using cosmic strings.

In this paper we will mainly use solutions of Einstein equations with two different interpretations. The results we will find might have some applications to the theories (String theory, GUT, Superstrings, ...) which can produce defects. In the next section, we will first review the gravitational field of a gravitational hedgehog solution [18] arising from triplet of scalar fields. Then following [26], we see that the same gravitational field can also be constructed using an ensemble of straight radial cosmic strings intersecting at a central point. This solution is called the "string hedgehog" solution. To better understand such spacetimes, in section 3 and 4 we study possible matchings of this space-time to different interior or exterior space-times. Our main observation will be that when we match two different hedgehog solutions via a spherical shell, a stringy-sphere arises. We will also discuss under which conditions the shell, the exterior and interior regions satisfy some energy conditions. In the last section we perform more general matchings and realize that the shell will deviate from its stringy character.

2. Gravitational hedgehog and string hedgehog

2.1 Gravitational field of a global monopole (hedgehog)

The simplest model that gives rise to the hedgehog [18] is described by the Langrangian

$$L = \frac{1}{2}\partial_\mu \phi^a \partial^\mu \phi^a - \frac{1}{4}\lambda \left(\phi^a\phi^a - \eta^2\right)^2 , \tag{2.1}$$

where ϕ^a is a triplet of scalar fields, $a = 1, 2, 3$. The model has a global O(3) symmetry, which is spontaneously broken to U(1). The field configuration describing a monopole is

$$\phi^a = \frac{\eta f(r) x^a}{r} \tag{2.2}$$

where $x^a x^a = r^2$. In flat space the hedgehog core has size $\delta \sim \lambda^{-1/2}\eta^{-1}$ and mass $M_{\rm core} \sim \lambda^{-1/2}\eta$. Folloving [18] we can take $f(r) = 1$ outside the core and the energy momentum tensor becomes

$$T_{rr} \approx -T_{00} \approx -\frac{\eta^2}{r^2} , \qquad T_{\theta\theta} \approx T_{\phi\phi} \approx 0 . \tag{2.3}$$

Let us consider the general metric

$$ds^2 = -A^2(r)dt^2 + B^2(r)dr^2 + C^2(r)d\Omega_2^2 . \tag{2.4}$$

Choosing the orthonormal basis one forms as

$$\begin{aligned} e^0 &= A(r)\, dt , \qquad & e^1 &= B(r)\, dr , \\ e^2 &= C(r)\, d\theta , \qquad & e^3 &= C(r) \sin\theta\, d\phi , \end{aligned}$$

for the nonzero components of the Einstein tensor for this metric one finds:

$$\begin{aligned} G_{rr} &= \frac{2A_rC_r}{AB^2C} - \frac{1}{C^2}\left(1 - \frac{C_r^2}{B^2}\right), && (2.5)\\ G_{\theta\theta} &= \frac{A_{rr}}{AB^2} + \frac{C_{rr}}{B^2C} - \frac{A_rB_r}{AB^3} + \frac{A_rC_r}{AB^2C} - \frac{B_rC_r}{B^3C} \\ &= G_{\phi\phi}\,, && (2.6)\\ G_{00} &= -\frac{2C_{rr}}{B^2C} + \frac{2B_rC_r}{B^3C} + \frac{1}{C^2}\left(1 - \frac{C_r^2}{B^2}\right). && (2.7) \end{aligned}$$

An immediate solution of (2.3) is found by choosing $A(r) = B(r) = \text{const.}$, $C(r) = \alpha r$. Then the metric becomes

$$ds^2 = -dt^2 + dr^2 + \alpha^2 r^2 d\Omega_2^2\,, \qquad d\Omega_2^2 = d\theta^2 + \sin^2\theta d\phi^2\,, \tag{2.8}$$

with the only nonzero components of the Einstein tensor satisfying

$$G_{00} = -G_{rr} = \frac{1-\alpha^2}{\alpha^2 r^2}\,.$$

This solution describes the asymptotic behavior of the exterior field of a global monopole outside the core and is first presented in [19] when studying spacetimes with angular deficit. Also, unlike straight cosmic strings, the exterior field of a gravitational monopole is not flat and it contains a solid deficit angle.

Notice that the metric (2.8) can be put into isotropic form

$$ds^2 = -dt^2 + (x^2 + y^2 + z^2)^{1-\beta}(dx^2 + dy^2 + dz^2)$$

with

$$\rho = \alpha r\,, \qquad \beta = \alpha^{-1}$$

and

$$(x, y, z) = \rho^\beta(\sin\theta\sin\phi, \sin\theta\cos\phi, \cos\theta)\,.$$

The general solution of the metric (2.4) satisfying (2.3) with $C(r) = r$ is given by [6]

$$A(r) = B^{-1}(r) = \left(1 - 8\pi G\eta^2 - \frac{2Gm}{r^2}\right). \tag{2.9}$$

Here m is a constant of integration and $m \sim M_{\text{core}}$ [18]. This solution is not asymptotically flat, see [20] for discussion of its ADM mass. It is shown in [21] and [22] that m may be negative and is responsible for tiny repulsive force of the core of the hedgehog. The dynamics and the stability of its core is discussed in [23, 24]. Notice that for large and positive m, the solution corresponds to a hedgehog swallowed by a black hole [18, 25]. Actually, we can neglect the mass term m for reasonable values of η and λ on the astrophysical scale [18]. When $m = 0$, the general solution (2.9) reduces to (2.8) after rescaling r and t coordinates with $\alpha^2 = (1 - 8\pi G\eta^2)$.

Thus so far we have a triplet of scalar fields ϕ^a of the form (2.2) generating gravitational field around the hedgehog (monopole). As shown in [26], the gravitational field around the hedgehog can also be constructed using an ensemble of radial cosmic strings.

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2.2 Gravitational field of a string hedgehog

Let us suppose that we have an ensemble of radially oriented cosmic strings, whose gravitational field is given in (1.1), all of them intersecting at a central point. If we take the strength of each string to be very small but the number of strings very big, then the resulting configuration will approximately have spherical symmetry. In the continuum limit, this symmetry will be exact. This is called "string hedgehog" [26]. The nonzero components of the energy-momentum tensor of this configuration will satisfy

$$T_{rr} = -T_{00}\,. \tag{2.10}$$

In this and the next section we will take $m = 0$. Thus, we will use the metric (2.8). Note that the energy contained inside a sphere of radius r_0 is given by

$$E = \int_0^{r_0} T_{00}\, e^1 \wedge e^2 \wedge e^3 = 4\pi(1-\alpha^2)r_0\,, \tag{2.11}$$

where $e^1 = dr$, $e^2 = \alpha r d\theta$, $e^3 = \alpha r \sin\theta d\phi$, and is linear in the proper radius of this sphere.

It is shown that we can have solutions with radial strings intersecting from a common point with [27, 28] or without [29, 30] a black hole at the center. It is also mentioned in those solutions that if mass per unit length of the strings (which is proportional to their angular deficit) is very small then we can go to the continuum limit. This actually corresponds to the situation we consider in this section.

3. Stringy hedgeball

In this section we match this string hedgehog solution to another hedgehog solution with the parameter β. When $\alpha = 1$ or $\beta = 1$, the interior or the exterior regions have flat Minkowski metric. Taking the exterior region minkowskian may sound unphysical, but, if we only consider the limits where $m = 0$ and $\alpha^2 \approx 1$ ($\eta^2 \ll 1$) than for large r the exterior region becomes almost minkowskian. Thus, we consider the solution with Minkowski exterior valid in these limits.

As we have emphasized before, for the string hedgehog solution, the energy density and the radial pressure ρ_r is vanishing at ∞. In this section we will try to find out the answer of this question: Is it possible to match this spacetime to another string hedgehog solution with different parameter such that both the interior and the exterior regions and also the shell satisfy certain energy conditions? Since for exterior and interior regions we will have different radial pressures, from continuity, we cannot smoothly match these two regions. Thus, we need a surface layer (infinitely thin shell) at the boundary of these two regions.

We take the interior metric as (2.8) and the exterior metric as

$$ds_+^2 = -dt^2 + d\rho^2 + \beta^2\rho^2 d\Omega_2^2\,.$$

There are several methods to calculate the energy momentum tensor of the shell [31]–[36]. Here we use an alternative method [36] which is equivalent to the standard method presented by Israel [31]. We also follow Lichnerowitz boundary conditions for surface layers [33]. The metrics interior and exterior to the hypersurface seperating these two regions must be continuous everywhere, but their derivatives with respect to the radial coordinate may contain discontinuities which give rise to the energy momentum tensor of the shell. This is the condition to have a surface layer at $r = r_0$. To do this we choose $\rho = \rho(r)$ and using the boundary conditions given below we can make the metric continuous at $r = r_0$. From the continuity conditions of the metric, at $r = r_0$ we have:

$$\rho(r_0) = \frac{\alpha}{\beta} r_0 \, ; \qquad \rho_{,r}(r_0) = 1 \, .$$

Choosing $\rho(r) = ar + b$ and using boundary conditions one gets

$$\rho(r) = r + \left(\frac{\alpha}{\beta} - 1\right) r_0 \, .$$

Thus the exterior and interior metrics become

$$ds_+^2 = -dt^2 + dr^2 + \beta^2 \left(r + \left(\frac{\alpha}{\beta} - 1\right) r_0\right)^2 d\Omega_2^2 \tag{3.1}$$

$$ds_-^2 = -dt^2 + dr^2 + \alpha^2 r^2 d\Omega_2^2 \, . \tag{3.2}$$

With the help of the Heaviside step function we can combine both the interior and the exterior metrics in the form

$$ds^2 = \theta(r - r_0) ds_+^2 + \theta(r_0 - r) ds_-^2 \, .$$

After calculating the Einstein tensor for this metric, the terms proportional to Dirac delta function will give the energy momentum tensor of the shell and the terms proportional to the step functions give the interior and the exterior solutions. Thus, the energy-momentum tensor for the whole spacetime can be expressed as

$$T_{\mu\nu} = T_{\mu\nu}^{(-)}\, \theta(r_0 - r) + T_{\mu\nu}^{(+)}\, \theta(r - r_0) + T_{\mu\nu}^{(0)}\, \delta(r - r_0) \, , \tag{3.3}$$

where

$$T_{\mu\nu}^{(k)} = \mathrm{diag}\left(\rho^{(k)}, p_i^{(k)}\right) , \qquad k = \pm, 0 \, ; \quad i = r, \theta, \phi \, . \tag{3.4}$$

Calculating the Einstein tensor, one gets for the nonzero components

$$\rho^{(-)} = -p_r{}^{(-)} = \frac{1 - \alpha^2}{\alpha^2 r^2} \, , \tag{3.5}$$

$$\rho^{(+)} = -p_r^{(+)} = \frac{1 - \beta^2}{\beta^2 r^2} \, , \tag{3.6}$$

$$\rho^{(0)} = \frac{2(\alpha - \beta)}{\alpha r} \, , \qquad p_\theta^{(0)} = p_\phi^{(0)} = \frac{(\beta - \alpha)}{\alpha r} \, . \tag{3.7}$$

So, we have matched a string hedgehog space-time to another string hedgehog solution via a surface layer at the boundary. Since we do not want to discuss the solutions where $\alpha^2 = 1 - 8\pi G\eta^2$ is negative, we limit the range of the parameters α and β as $-1 \leq \{\alpha, \beta\} \leq 1$ since we have $0 < \{\alpha^2, \beta^2\} \leq 1$.

For an energy momentum tensor of the form (3.4) we have the weak energy condition ($\rho \geq 0, p_i \geq 0$), the dominant energy condition ($\rho \geq |p_i|$) and strong energy condition ($\rho + \sum_i p_i \geq 0$) [37]. Thus, the weak energy condition is not satisfied since p_i is negative. When α and β have the same signs, to have a positive energy density, we need $|\alpha| > |\beta|$. When they have different signs the energy density is always positive. The shell has the equation of state:

$$\rho^{(0)} = -\left(p_\theta^{(0)} + p_\phi^{(0)}\right) . \tag{3.8}$$

Then, when the shell has positive energy density, it satisfies dominant and strong energy conditions. Note that both the interior and the exterior regions also satisfy these energy conditions for these ranges of the parameters. The total energy of the shell is $E = 8\pi\alpha(\alpha - \beta)r_0$.

We can interpret this shell as a shell composed of uniformly distributed cosmic strings lying on the surface of a sphere since it satisfies (1.3).

Thus when we want to embed a stringy hedgehog spacetime to another hedgehog solution, a stringy spherical shell arises. For a certain range of the parameters, both the shell and the string hedgehog spacetime satisfies energy conditions.

If we take $\beta = 1$ the exterior region becomes flat Minkowski spacetime. In this case to have a shell satisfying energy conditions, we need $-1 < \alpha < 0$. We can call this solution with flat exterior a “stringy hedgeball” solution. Or, we can discuss the opposite situation. If we take $\alpha = 1$, then the interior region of the spherical shell becomes flat. For this case, to satisfy energy conditions, one needs $0 < \beta < 1$.

4. Generalizations and discussions

In this section we consider the interior and the exterior metrics $ds^2_\pm$ as hedgehog-Schwarzschild-de Sitter solutions since we can superpose the Schwarzschild-de Sitter solution with the hedgehog solution [26]. For the interior and the exterior regions we label the coordinates as (t, r, θ, ϕ) and (τ, R, θ, ϕ). The metrics are of the form (2.4) where the metric functions are given as

$$A_-(r) = B_-^{-1}(r) = \left(\alpha^2 - \frac{2m}{r} - \frac{\Lambda_-}{3} r^2\right)^{1/2} , \tag{4.1}$$

$$A_+(R) = B_+^{-1}(R) = \left(\beta^2 - \frac{2M}{R} - \frac{\Lambda_+}{3} R^2\right)^{1/2} , \tag{4.2}$$

$$C_-(r) = r \, , \qquad C_+(R) = R \, . \tag{4.3}$$

We require them to be continuous at $r = r_0$. We again keep the interior metric as it is and for the exterior metric we choose $R = R(r) = ar + b$ and $\tau = Et$. Then we have the following boundary conditions:

$$R(r_0) = r_0 \, , \qquad R_{,r}(r_0) = \frac{A_+(r_0)}{A_-(r_0)} , \qquad E = \frac{A_-(r_0)}{A_+(r_0)} . \tag{4.4}$$

Our exterior metric becomes

$$ds^2_+ = -A^2_+(r)dt^2 + A_+^{-2}(r)dt^2 + R^2(r)d\Omega_2^2 . \tag{4.5}$$

with

$$A_+(r) = \sqrt{\beta^2 - \frac{2M}{R(r)} - \frac{\Lambda_+ R^2(r)}{3}}\, \frac{A_-(r_0)}{A_+(r_0)}\,, \tag{4.6}$$

$$R(r) = \frac{A_+(r_0)}{A_-(r_0)} r + \left(1 - \frac{A_+(r_0)}{A_-(r_0)}\right) r_0\,. \tag{4.7}$$

The energy momentum tensor of the shell has the following components

$$\rho^{(0)} = 2K(r_0)\,, \qquad p_\theta^{(0)} = p_\phi^{(0)} = -K(r_0) + L(r_0)\,, \tag{4.8}$$

where

$$K(r_0) = \frac{A_-(r_0)}{r_0}(A_-(r_0) - A_+(r_0)), \tag{4.9}$$

$$L(r_0) = -\frac{A_-(r_0)}{A_+(r_0)}\left(\frac{\Lambda_+ r_0}{3} - \frac{M}{r_0^2}\right) + \left(\frac{\Lambda_- r_0}{3} - \frac{m}{r_0^2}\right). \tag{4.10}$$

This solution is quite general since choosing some parameters zero we can recover the more simple solutions such as when $m, M, \Lambda_\pm$ vanishing, the solution reduces to the stringy shell solution we have presented in the previous section. If $\Lambda_\pm, = 0, \alpha = \beta = 1$ we have a shell around a black hole [38].

Note that this shell no longer satisfies (3.8). However, whenever the parameters other than α, β go to zero, $L(r_0)$ also goes to zero and we recover (3.8). The presence and the difference of these parameters deviates the shell from its stringy character. For this general case we can specify an equation of state for the shell $p_i^{(0)} = p_i^{(0)}(\rho^{(0)})$ and solve for the parameters since for this case we have more parameters than the equations. We can even have a domain wall satisfying $p_\theta^{(0)} = p_\phi^{(0)} = -\rho^{(0)}$ if the parameters satisfy $K(r_0)+L(r_0) = 0$ or a shell of photons satisfying $p_\theta^{(0)} = p_\phi^{(0)} = \rho^{(0)}/2$ if the relation $4K(r_0) - L(r_0) = 0$ holds.

In this paper using the thin shell formalism, we have glued some interior solutions in spherical coordinates to an exterior solution via a spherical shell where the interior and the exterior solutions have the same character with different parameters.

After reviewing the global hedgehog (monopole) and the string hedgehog solutions, we have first chosen the interior and the exterior regions as the string hedgehog solution which can be made up with an ensemble of radial cosmic strings and we have calculated the energy momentum tensor of the shell seperating these two regions. We see that this infinitely thin shell is composed of cosmic strings lying on the surface of the sphere. For certain ranges of the parameters, the interior, the exterior and the shell satisfy the dominant energy condition.

Then we have chosen the interior and the exterior regions as hedgehog-Schwarzschild-de Sitter solutions with different parameters, and we have seen that for this case the shell is no longer stringy.

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